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Financial Time Series: Applying and Analyzing Time Varying Volatility Models in the Cryptocurrency Bitcoin

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Abstract

What is Bitcoin and how does it work? We explore the popular cryptocurrency using time series analysis. We demonstrate how one can model the returns and volatility of the asset with GARCH models. We then pose a discussion of what the future of Bitcoin is, its identity as a so called "currency" and its evolving identity as a financial asset.

1. Introduction

The endless arrival of both data and information in financial markets demands the analysis of volatility or the variation in asset prices across time. Volatility is of upmost importance for financial researchers, practitioners, and risk adverse investors. The unpredictability of prices negatively affects investors, and also impacts "consumption patterns, corporate capital investment decisions, leverage decisions and other business cycles and macroeconomic variables[1]." Tools for modeling such processes can begin with financial time series, which investigate changes in asset returns and volatility across time. Stylized facts or typical exhibited behaviors and properties in financial data play an important role when it comes to modeling conditional volatility of asset returns. The unconditional distributions tend to have fat tails and are accompanied by changing volatility over time, where high volatility periods are followed by low ones and vice versa. We describe such behavior as time varying conditional variance or volatility clustering[1]. Practices in financial time series have not only aimed to model fluctuations in returns, but capture volatility clusters and the impact of good and bad news on the volatility of asset returns. Much research is built upon initial work of Engle, who developed the autoregressive conditional heteroskedasticity model, or more commonly known as the ARCH model[2]. The GARCH model shortly followed, or the generalized autoregressive conditional heteroskedasticity model, an extension of the ARCH model by Bollerslev[3].

Further investigation of the volatility of asset returns has been carried out with GARCH models since their induction, as well as various model extensions. Some of which include the exponential GARCH or EGARCH model introduced by Nelson[4], the asymmetric power GARCH or APGARCH introduced by Engle and Ng[5], the GJR-GARCH introduced by Glosten, Jagannathan, and Runkle[6], the quadratic GARCH or QGARCH introduced by Sentana[7], the regime switching GARCH or RSGARCH introduced by Cai[8], Hamilton and Susmel[9] and Kim and Kim[10] and Susmel[11], the threshold GARCH or TGARCH, developed by GJR[6] and Zakoian[12]. Extensions of the GARCH model aim to capture asymmetry of volatility due to news impacts, as well as strengthen forecasts of conditional volatility. A characteristic of the volatility of returns is asymmetry, which is commonly referred to as leverage effect. Asymmetry in volatility occurs due to larger impacts from negative shocks, as opposed to positive shocks, where shocks are the results of good and bad news[13]. Previous examination of asymmetric volatility attributes the leverage effect to the causal relationship between bad news, lower prices, and higher leverages[14]. Motivated by past developments with GARCH models of exchange rates and stock prices, this paper will aim to apply them to the cryptocurrency Bitcoin.

Bitcoin is a virtual currency that is decentralized, peer to peer, and has encrypted transactions. Transactions of the cryptocurrency involve no centralized authority, clearing house, or institution. Bitcoin operates with block chain technology, of which a transparent and secure system of accounting is used, that transfers ownership and stored data of every Bitcoin transaction via the above mechanisms[15]. Bitcoin has received much criticism, scrutiny, and media attention since its release, leading to high amounts of volatility in its life cycle thus far.

Previous empirical investigations of Bitcoin have found it is more characteristic of an asset rather than a currency, and also possessive of risk management and hedging capabilities[16]. Examination of volatility has also taken place, with multiple univariate GARCH[17] models and multivariate GARCH models, comparing Bitcoin with other assets such as gold[16]. What has not been extensively investigated is the exploration of the leverage effect in the volatility of Bitcoin returns. This paper will aim to employ methodologies of Beg and Anwar[18] and Zivot[14] to explore various univariate GARCH models that measure the effects of good and bad news on Bitcoin returns.

2. Data and Methodology

The data used for this paper spans from August 21, 2013 to August 18, 2017. Bitcoin to dollar exchange rate data has been sourced from Coindesk[19]. The data span 998 observations after the removal of holidays and weekends from all data points, thus following a steady stream of financial events and one that is similar to other exchange rates[20]. Preliminary examination of the data confirm GARCH effects, with the log return and squared return series exhibiting clusters of volatility. We let p_t denote the price of Bitcoin at time *t*. We define the return x_t to be the natural log of the argument:

$$x_t = ln\frac{p_t}{p_{t-1}} \quad (1)$$

We present time series plots below of both the log returns and the squared log returns.

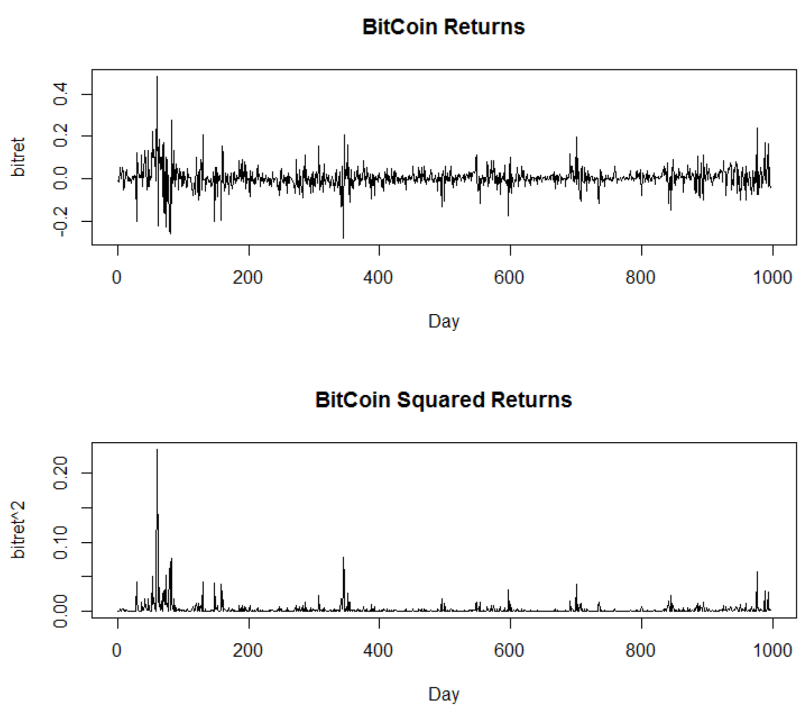

Figure 1. Time series plots of Bitcoin log returns and squared log returns.

Initial testing of Bitcoin returns indicates asymmetry. Kurtosis and skewness test statistics $\sqrt{b_1}$ and b_2 were used, as seen in Shapiro and Wilk[21]. Additionally, these tests both agree with the Jarque Bera test of normality, a joint test of skewness and kurtosis of a normal distribution, with significance at the 5% level. The test of zero mean was significant at the 5% level. These results indicate Bitcoin returns are non-normal with pronounced kurtosis[18]. Further, significant skewness at the 5% level indicates returns are also heavily skewed and leptokurtic. We present the following table below, summary statistics and hypothesis tests of Skewness, Kurtosis, and the Jarque Bera test for normality.

Table 1. Summary statistics and preliminary tests of Bitcoin returns.

Summary Statistics				
Series	Mean	Std. Deviation	Minimum	Maximum
x	0.0036	0.0519	-0.2809	0.4848
Preliminary Tests				
Test for	Test Value		p- value	
Mean = 0	2.2064		0.0276	
Skewness = 0	0.631		0.0000	
Excess Kurtosis = 0	15.791		0.0000	
Jarque Bera (JB)	6863.10		0.0000	

Following this examination, we begin fitting the mean process of the return series. We define the return series as the following:

$$\boldsymbol{x_t} = (x_1, \dots, x_t) \quad (2)$$

Where the return series is an $(N-1)$ x 1 vector, due to calculating log differences from one period to the next in Eq. (1). Prior to fitting the conditional mean process, the return series was tested for stationarity, with the Augmented Dickey-Fuller test of unit root. Following the procedure of Ng and Perron[22], we specify the lag length of the test as $p = 4$. We define the unit root test as the evaluation of the following regression model:

$$\Delta x_t = a_0 + a_1 x_{t-1} + \sum_{k=1}^{p} \beta_k \Delta x_{t-k} + \xi_t \quad (3)$$

The results of the test are presented in the table below.

Table 2. Results of Augmented Dickey-Fuller test of unit root.

Unit root test	Test value	p- value
H_0: $a_1 = 0$	τ_μ = -12.283	0.0000
H_a: $a_1 < 0$		

We reject the null hypothesis that a unit root is present in the series. It follows that the Bitcoin log returns are characteristic of a stationary series. The conditional mean model can be specified as a function of the return series, and follows as a combination of both autoregressive terms (AR (p)) and moving average terms (MA (q)). Combination of these two processes has famously been constructed by Box and Jenkins[23]. Following the presence of a stationary series, one can specify an autoregressive moving average (ARMA) model for the conditional mean as follows:

$$x_t = \mu + \sum_{i=1}^{p} \varphi_i x_{t-i} + \sum_{j=1}^{q} \lambda_j v_{t-j} + v_t \quad (4)$$

Where x_{t-i} is the return at i time periods ago with order p and v_{t-j} is the residual at j time periods ago with order q, and v_t is the residual at time t and assumed to be normal with mean zero and constant variance h_t. The ARMA model for the Bitcoin return series was found to be an ARMA(4,1), following minimization of Akaike Information Criteria with $v_t \sim t(0, h_t)$. That is, distributed as a Student t, with mean zero and constant variance h_t given the fat tails of the Bitcoin log return series. The results of the conditional mean specification are presented in the table below:

Table 3. Estimates of the ARMA(4,1) conditional mean model.

ARMA(4,1) model with Intercept			
Parameter	Estimate	Std. Error	p-value
μ	0.0021	0.0001	0.0272
AR(1)	-0.7694	0.1141	0.0000
AR(2)	0.0243	0.0536	0.6499
AR(3)	0.1228	0.0485	0.8000
AR(4)	0.0309	0.0386	0.4228
MA(1)	0.7767	0.1059	0.0000

Further examination of the conditional mean model was carried out with the Weighted Ljung-Box test[24] on the standardized residuals. This test evaluates autocorrelation in the residuals of a fitted mean or variance model. It has been shown by Fisher and Gallagher to have higher power when compared to other Portmanteau tests, as well as greater stability at lags close to the sample size[24]. The null hypothesis assumes the data (residuals) have no serial correlation versus an alternative hypothesis of serial correlation. The presence of autocorrelation in the residuals of the conditional mean model confirms time dependence amongst lags. Therefore, significant p-values of the test confirm the existence of nonlinearity, or GARCH effects in the Bitcoin log returns, and agree with earlier examination the Figure. 1. We present the Ljung-Box statistics of various lags in the table below. Where LBQ (x) denotes the lag at time period x:

Table 4. Results of Weighted Ljung-Box Statistic

Ljung-Box Statistics		
Weighted Standardized Residual	Test Value	p- value
LBQ (1)	1.386	0.2391
LBQ (14)	9.547	0.0001
LBQ (24)	13.272	0.3609
Weighted Standardized Squared Residuals		
LBQ (1)	0.0205	0.8862
LBQ (5)	0.7373	0.9156
LBQ (9)	1.1166	0.9809

Following the above examination and mean model specification, two GARCH models are fitted to the Bitcoin log return series to capture nonlinear dynamics in the variance function[18]. The standard GARCH[3] is parsimoniously defined as follows:

$$v_t | I_{t-1} \sim iid(0, h_t) \tag{5}$$

$$v_t = z_t \sigma_t \tag{6}$$

$$h_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j v_{t-j}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{7}$$

Where I_{t-1} is the information set available at time $t-1$. Observe v_t is the residual in Eq. (4). Additionally, α_j and β_i are the conditional variance parameters, and are restricted such that $\alpha_0 > 0$, $\alpha_j \geq 0$ and $\beta_i \geq 0$. Also observe that v_t is the product of z_t and σ_t , where z_t is a sequence with zero mean and unit variance and σ_t is the conditional standard deviation at time t[13]. In an effort to capture volatility asymmetry the estimation of two different GARCH models was conducted; the exponential GARCH (EGARCH) and the traditional GARCH. The EGARCH model captures asymmetry or the leverage effect with the addition of γ_j as a parameter. Additionally, this model is a transformation of the GARCH(1,1), where the logarithm of the conditional volatility h_t is modeled[13]. This guarantees conditional volatility is positive and does not restrict coefficients in the model to those of Eq. (7). Additionally, note that h_t is the conditional variance (volatility) at time t and can also be written as σ_t^2.We represent the EGARCH model as follows[4]:

$$h_t = log\sigma_t^2 = a_0 + \sum_{j=1}^{q} \alpha_j \frac{|v_{t-j}| + \gamma_j v_{t-j}}{\sigma_{t-j}} + \sum_{i=1}^{p} \beta_i h_{t-i} \quad (8)$$

Where, we define "good news" when v_{t-j} is positive. Thus, we have a total effect of $(1+\gamma_j)v_{t-j}$. It follows that v_{t-j} is negative when there is "bad news". Thus, we have a total effect of $(1-\gamma_j)|v_{t-j}|$. It is expected that γ_j would be negative, given that more impactful shocks are the result of bad news, as discussed above[13].

Amongst the two models, we specify student t residuals with 4 degrees of freedom following minimization of Akaike Information criteria, and a first order lag for both the residual and volatility. Resulting in GARCH (1, 1) and E-GARCH (1, 1) respectively. All parameters were estimated with quasi-maximum likelihood estimation[18] in R programming software with the "rugarch" package[25].

3. Empirical Results

We present our findings below, with estimates of both identified GARCH models:

Table 5. GARCH(1,1) and EGARCH(1,1) estimates.

Volatility Models			
GARCH(1,1)	Estimate	Std. Error	p- value
α_0	0.00006	0.000019	0.0007
α_1	0.16695	0.037440	0.0000
β_1	0.76563	0.030727	0.0000
EGARCH(1,1)			
α_0	-0.28632	0.081625	0.0004
α_1	0.01195	0.019464	0.5391
β_1	0.95214	0.012600	0.0000
γ_1	0.30883	0.036579	0.0000

The results of the GARCH(1,1) model illustrate significance of the model at the 5% level, indicating a good fit for this model. We also observe a high persistence in shocks, with $\alpha_1 + \beta_1 = .973$. This indicates that shocks are actually persisting with resultant volatility from good news effects[18]. The EGARCH(1,1) model presents an insignificant residual term, but a significant volatility term and asymmetry term, with significance at the 5% level. Examination of model fits are presented in Figure. 2 as well as the news impact curve suggested by previous examinations of Engle[26] and Pagan[27]. The news impact curve plots the relationship between the conditional variance at time t and the error term at time $t-1$ as seen in Figure 2. In the presence of volatility asymmetry, the plot would exhibit skewness to the right. In the case of Bitcoin log returns, it follows a nearly symmetric shape, with slightly greater responses from positive shocks.

4. Conclusion

The above analysis successfully examined clusters of volatility in daily Bitcoin log returns, or the presence of GARCH effects. A conditional mean model was then specified, followed by the examination of the traditional GARCH and exponential GARCH models. Our results illustrate that Bitcoin log returns do exhibit clusters of volatility and can be modeled effectively with conditional volatility models. Further, leverage effects were determined to not exist in the log return series. That is, negative shocks do not asymmetrically affect the volatility of Bitcoin log returns although positive ones are more representative in the examined sample. A significant positive asymmetry term γ_1 indicates that Bitcoin returns are not asymmetrically impacted by the presence of bad news[16]. That is, larger volatility has not been found to be associated with negative shocks. However, a positive asymmetry term suggests positive shocks are more frequent. Further, that lower volatilities are associated with such news given that market participants aim to keep their current positions[20].Previous examination has encouraged the use of Bitcoin to hedge market risks due to these properties[17]. A further examination of Bitcoin returns are bound to take place in the future. Perhaps the analysis of intra-day data would present different results due to the 24hr availability for trading of the asset. Further, monthly data could be examined for leverage effects as more data become available.

The cryptocurrency Bitcoin presents fascination for most, in use and in concept. The analysis performed in this paper has demonstrated how one can model the volatility of the asset and aims to motivate further analysis of Bitcoin as more data become available. The future of Bitcoin may appear unrealistic for many, although the asset has shown thus far to be a useful financial asset.

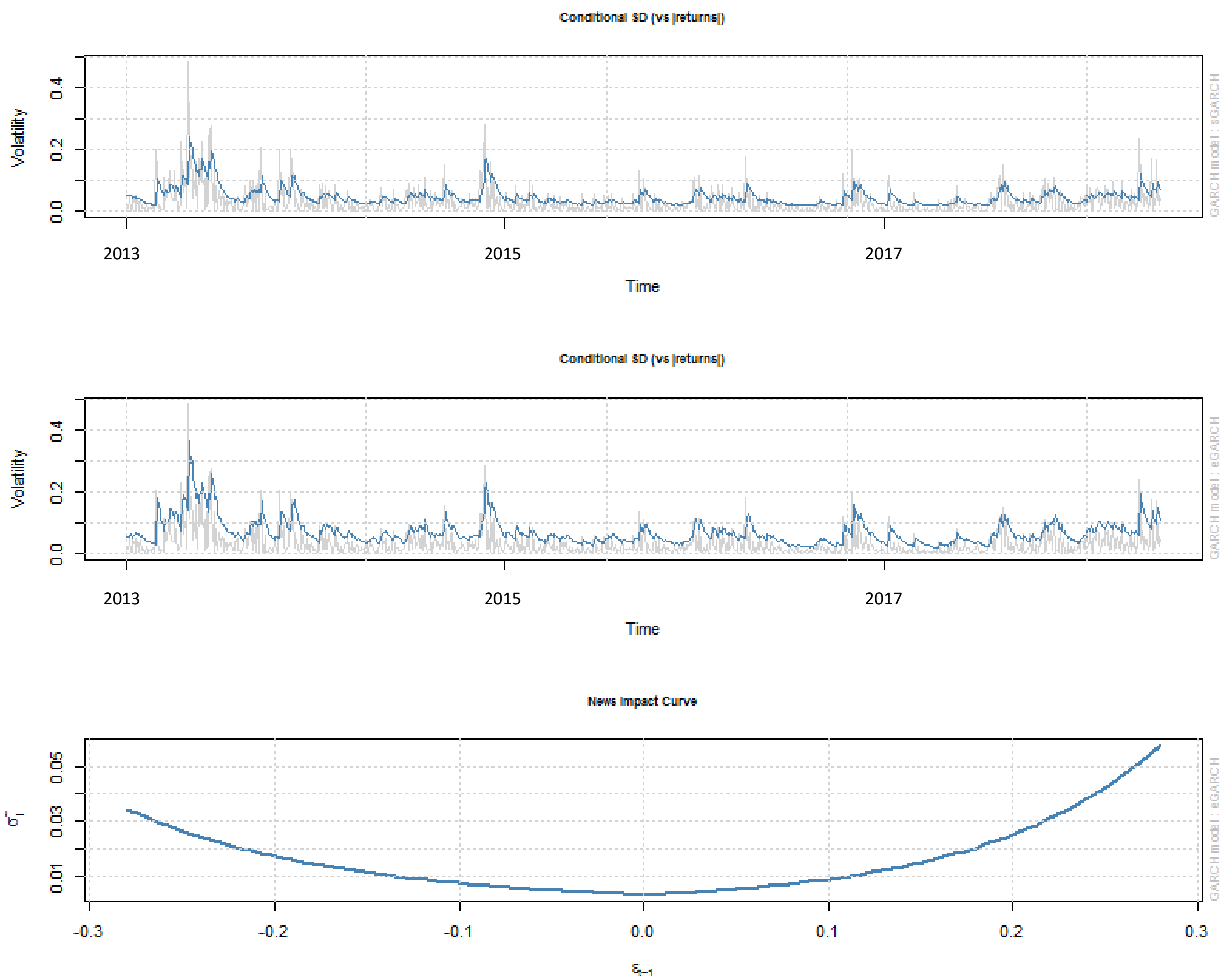

Figure 2. Model fits of GARCH(1,1) and EGARCH(1,) and news impact curve of EGARCH(1,1) model.

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