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Comparison of Theoretical Methods for Measuring the Speed of Sound in Tubes with Varying Radii

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Abstract

The speed of sound in air is typically known for any given temperature and can be verified in a tube with a constant radius by measuring the resonant frequencies which are a harmonic sequence that is linearly related to sound velocity. We can measure those frequencies using a microphone, sound generator, and lock-in amplifier. However, when this relationship is used, the speed of sound found is much lower than the speed measured when sound travels through a tube with varying radii. This indicates to us that our equation needs to be altered to compensate for the varying radii. By testing a variety of tubes of different length and radius we can compare different theoretical methods and determine which method is best.

1. Experimental Method

To gather the required data for our experiment we used a sound generator, microphone, lock-in amplifier, Pasco SW750 computer interface module, and Capstone software. The lock-in amplifier is a crucial component to the experimental set up because this device detects and extracts a specific signal at a specific frequency. In doing so it eliminates much of the electrical noise giving much more accurate data. Using this device in conjunction with Capstone, the frequencies being produced can be modulated in such a way that they increase with time. With the microphone taking in those frequencies and plotting them on a graph, over time the harmonics will be found on this graph as a peak in the data.

2. Theory And Data

The speed of sound in air is dependent on the temperature *t*, and can be found by using the equation:

$$v_{air} = 331.3\sqrt{1+\frac{t}{273.15}}\frac{m}{s} \tag{1}$$

The accuracy of this equation can be verified by using a straight tube of constant radius. This is done by measuring the harmonic frequencies that a tube produces when sound is traveling through them. Once those harmonics are detected the relationship below can be used to calculate the velocity of sound, *v*.

$$f_{harmonic} = \frac{v}{2L} \quad (2)$$

where n is the node and L is the length of the tube in use. With some rearranging the relationship will then become:

$$v = 2Lf \quad (3)$$

From here the calculated velocities can be compared to the known velocity found using equation (1) at some temperature. This comparison will determine how accurate the theoretical relationship is. When these calculations are done with the data collected, the values of v found are much lower than that of the known v at some temperature. This is a strong indicator that the theoretical model in use needs to be edited in such a way that it produces results closer to the known v found with equation (1). This can easily be proven when taking into consideration a simple straight tube of constant radius. When using the above relationship, we find that v is much lower than the known v. Now if a simple length correction is added to the above relationship we get:

$$v = 2fL(1 + \frac{1.22r}{L}) \qquad \text{Note: let } L' = \left(1 + \frac{1.22r}{L}\right)L \quad (4)$$

where r is the radius of the tube. This correction can be added because sound is a compression wave and when it travels through a tube, small regions of air become stagnate at each of the openings. This stagnate air effectively increases the overall length of the tube. Using this new equation, the produced results become much closer to the known value of v.

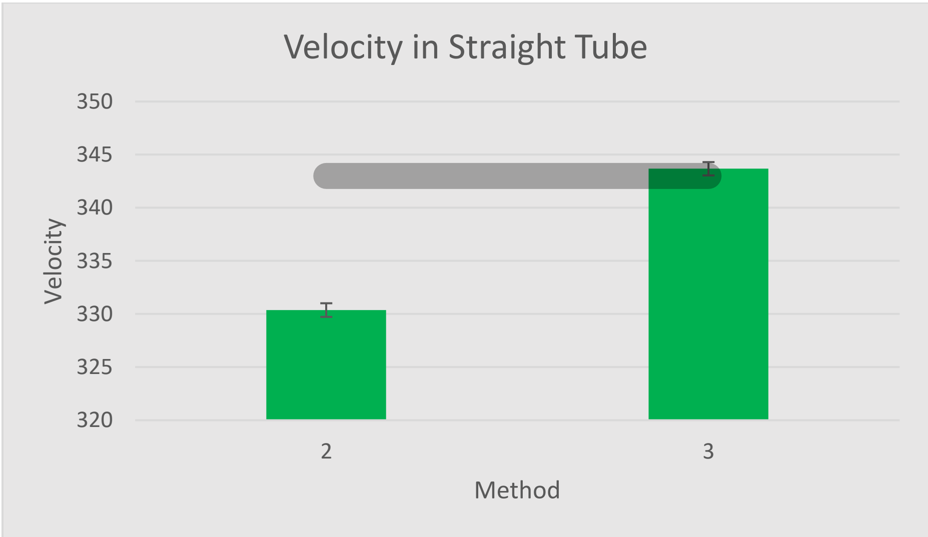

Figure 1 Comparison Between Equation (3) and (4) respectively. Method 2 is equation (3) and method 3 is equation (4). The gray strip is v from equation (1)

The results of equation (3) and (4) become even less accurate when considering a tube of varying radii. Since there are varying dimensions throughout these types of tubes, we must consider each dimension in our calculations. This becomes apparent when we compare the harmonics of both a corrugated and straight tube of similar length.

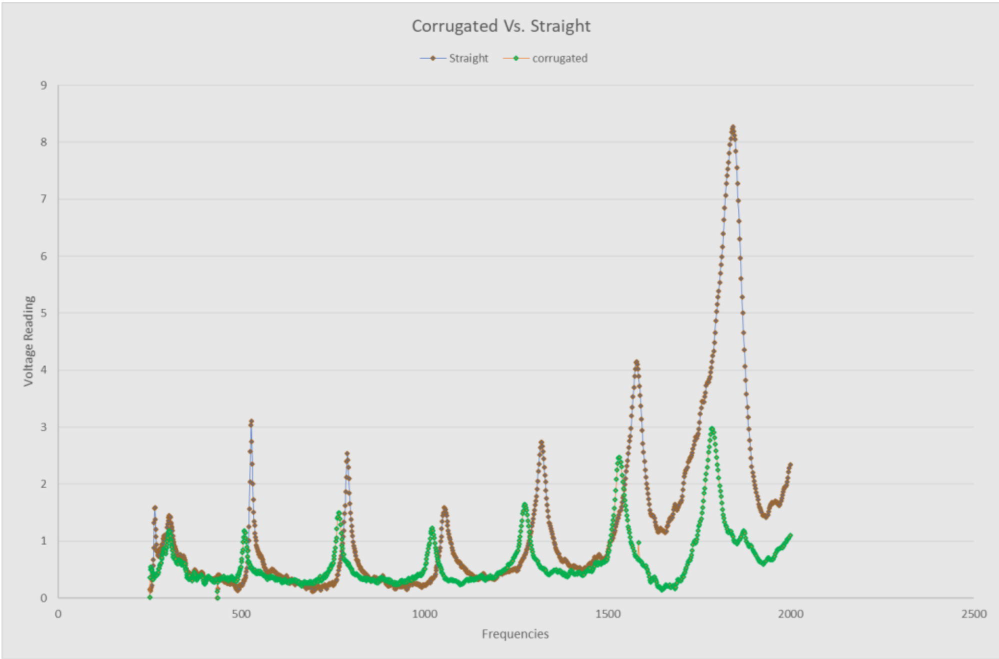

Figure 2 Comparison Between Straight And Corrugated Harmonics

It is apparent from the graph that the harmonics for the corrugated tube occurs at a lower frequency than that of the straight tube. There are different parameters that can be added to equation (4) that can help account for the varying radii. When this parameter is accounted for equation (4) becomes:

$$v = \frac{2fL'}{\sqrt{\frac{V_{in}}{V_{out}}}} \tag{5}$$

where V_{in} and V_{out} are the inner most and outer most volume respectively. This approximation produces closer results to the known speed, but better results can be obtained when each dimension of the corrugations are taken into consideration specifically[1]. When this is considered, the equation then becomes:

$$v = 2L'f\left(\sqrt{1+\frac{2wh}{rp}}\right) \tag{6}$$

In this last equation the corrugation separation, height of the corrugation, and the width of each corrugation are taken into consideration. It is vital to get accurate measurements for each dimension, so an average of multiple measurements was used for the final calculations. In this last graph we can see how each of the different equations compare to one another, and how close each one is to getting to the known value of v.

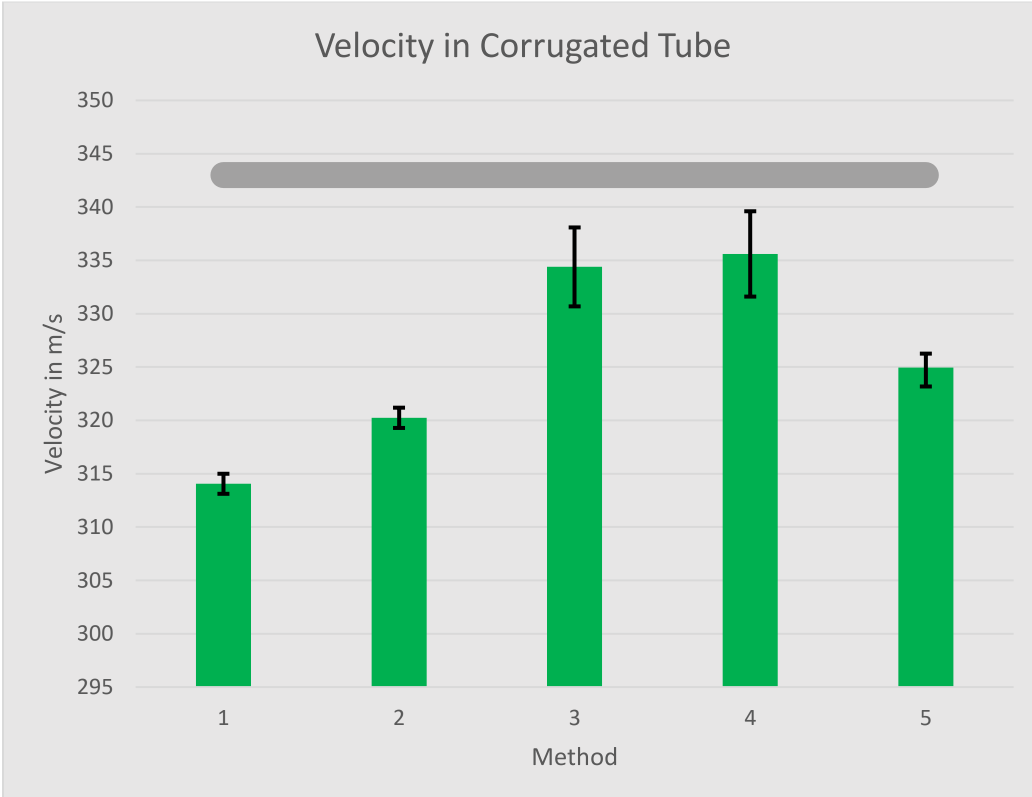

Figure 3 Comparison Of All Methods

3. Conclusion

From the data collected it is clear to see that corrugations effect the speed of sound traveling through a tube. By editing a known equation for harmonic frequencies, better predictions of the speed of sound can be made as it travels through a tube. Finally, higher accuracy can be achieved by applying analytic solutions to a cosine squared tube model.

4. Acknowledgements

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5. References

1. Michael J Ruiz and James Perkins, "The Monster Sound Pipe," *Physics Education*, volume 52, no.2 (2017).