Mathematics and Sports

Technical and Program Component scores frozen together: Difficulty bias and outcome prediction in international figure skating

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Abstract
Ideally, technical and non-technical scores in judged sports would be independent. Any relationship between technical and non-technical scores would indicate that these scores are dependent upon each other. The International Judging System (IJS) was introduced by the International Skating Union in 2003 as a way to remove some of the highly publicized biases that were present in the previous judging system. In this article, we describe an unexpected linear relationship between Technical Element and Program Component scores in all four figure skating disciplines at 2018-2019 season competitions hosted by US Figure Skating and the International Skating Union. These relationships imply difficulty bias within the scoring system and the possibility of outcome prediction of the Program Component scores based on the Technical Element scores. Furthermore, we also illustrate that scoring across technical elements effectively renders jumps as the most important element in singles and pairs skating. These findings have profound implications for sport governance and can be used for strategic advantage.
Keywords: Difficulty bias, Judging bias, outcome prediction, performance quantification

1 Introduction to quantitative scoring of sport performances

Many Olympic sports involve quantitative evaluations of qualitative performances. For example, athletes receive a numeric score for performances in the summer sports of boxing, diving, synchronized swimming, and gymnastics, and the winter sports of ski jumping and figure skating. In an ideal scenario, when scores are awarded for both technical and non-technical aspects of the same sports performance, the two aspects would independently contribute to the overall outcome.

The International Skating Union (ISU 2003) hoped to develop a judging system that rewards skaters who have a balance of technical and artistic components. In a 2003, "Frequently Asked Questions" document, the ISU explained the rationale for having a scoring system that has separate and transparent rubrics for rating technical and non-technical aspects of performances. They explained that the purpose of the International Judging System (IJS) was to prevent competitions from turning into "jumping contests (p. 8)." Furthermore, the ISU noted that their "Well-Balanced Program" was meant to protect the competitions from becoming jumping contests by limiting the number of jumps available to skaters.

There are three main contributions in this paper that distinguish it from previous literature on sports scoring. First, it updates the research on the IJS. Most IJS studies were performed soon after its inception in the 2004-2005 season. However, the ISU refines the scoring system after each season to better reflect its intentions. Athletes and coaches might hope that the scoring system would rate skaters' technical and program component scores as independent entities after 15 years of revisions, including 4 Winter Olympic Games within this time span. The second contribution of this paper is that it reveals a linear correlation within all of the figure skating disciplines, whereby the more artistically-based score depends on the technical score. The third contribution of this paper is that it provides an example of how to use this linear relationship for strategic decisions within skating.

1.1 Known biases within multiple sports' scoring systems

There is a plethora of statistical documentation showing that biases exist within subjective sport ratings. For a more comprehensive summary of all biases, see the literature review conducted by Boen et. al. (2008). In this section, we provide three examples of similar biases that were present in multiple sports including figure skating.

Judges in performance sports typically have an open feedback system whereby they watch a performance, submit their own scores, and then learn what the average score of the entire judging panel was for the same performance. Researchers found a conformity bias, whereby gymnastics judges adapted their scoring to the scores of their colleague judges (Boen et. al., 2008). In a similar study, Vanden Auweele et. al. (2004) examined synchronized swimming data and found that knowing how other judges

scored a performance influenced later scores in the competition. In effect, this knowledge encouraged judges to try to deviate less from the mean scores on future performances. This conformity bias was also observed in figure skating world championship results from 2001 through 2003 by Lee (2008), who hypothesized that judges' have an incentive for not issuing outlier scores because of the selection process used by the national skating federations. Judges whose scores have a large deviation from other judges' scores do not get assigned to judge future competitions.

A nationalistic bias was identified in diving competitions at the 2000 Summer Olympic Games (Emerson, Seltzer & Lin, 2009) and in figure skating results from four Winter Olympic Games from 1976 to 1994 (Campbell & Galbraith, 1996), whereby judges representing one country favor athletes representing the same country. Likewise, an analysis of selected international figure skating competition results from 2005 to 2018 revealed that a nationalistic bias was present, but this bias did not significantly impact the overall placements (Zhu, 2018). In a large scale study of Winter Olympic Games results from 1948 to 2002, a political bias was identified, whereby judges' ranking of skaters seemed to be biased based on whether the skater was from an allied or enemy county in the international political realm (Sala, Scott & Spriggs, 2007).

In gymnastics, a technical score is awarded to rate the difficulty of each element an athlete performs. A separate execution score is also awarded for the performance, rating how well the athlete performs each technical element. The technical and execution scores are intended to be independent through the design of the scoring system. However, Morgan and Rotthoff (2014) and Rotthoff (2020) found within their data sets (the 2009 and 2013 World Artistic Gymnastics scores, respectively) that athletes who performed more difficult technical elements also received higher execution scores, regardless of how well the athletes performed the technical elements. For example, the mere attempt of a difficult dismount sometimes yielded a higher execution score than a less difficult dismount performed perfectly. This bias towards technical difficulty rewarded athletes with ambitious routines in two judging categories, thereby perhaps unintentionally altering the sport.

Given that similar biases have been observed between the judging of sports as diverse as diving, synchronized swimming, gymnastics, and figure skating, it is natural to question whether similar biases are also present in figure skating. In this paper, we will demonstrate that a similar difficulty bias exists in skating as exists in gymnastics. By analyzing the performance evaluations of the 2018-2019 skating season, we found that technical scores were not judged independently from artistic scores. Instead, they could regularly be used to predict artistic scores. In addition, we will demonstrate how this type of outcome prediction can be used by skaters to gain strategic advantage.

1.2 The International Judging System of figure skating scoring

Besides Zhu's (2018) analysis, most of the research on judging biases in skating were based on competitions that used the 6.0 System of judging, which was used until 2003 by the International Skating Union. In the 6.0 System, a group of judges rated each skater's performance in the areas of technical merit and presentation, which were numerical scores from zero to six that signified a skater's relative ranking against other skaters in their event. Researchers have revealed several judging biases within this 6.0 System. For example, because judges only had to give one holistic score for all of the technical elements, it was difficult to have consistency among judging decisions (Lock and Lock, 2003). In the pairs event at the 2002 Olympic Winter Games, the French judge rated the Russian team in first place, even though this team made several technical mistakes (ISU 2002). The 6.0 System had only one judging panel, and as such, it was possible for this individual judge's scores to significantly impact the overall result of the event (ISU 2002). After this highly publicized judging scandal occurred at the 2002 Olympic Winter Games, the ISU developed a new system for scoring at international figure skating competitions called the International Judging System (IJS).

In what follows, we summarize how IJS scores are determined, more fully described by ISU (2021a). Under the IJS, each skater's program earns a Total Segment Score (TSS) which is the sum of the Total Element Score (TES) and the Program Components Score (PCS). There are two independent panels rating the skating performances. The technical panel rates the difficulty of the technical elements, and a panel of judges rate the execution of the technical elements and the non-technical aspects of the overall performance.

The difficulty of the elements combined with the execution of the elements are factored into the TES. Thus, TES is a measurement of technical skating ability. Depending on the discipline, the technical elements will fall into the general categories of jumps, spins, step sequences, lifts, twizzles, choreographic elements, and death spirals. Meanwhile, the PCS (ISU 2021b) is a measurement of non-technical presentation quality as determined by scores assigned to certain program components, which are [1] skating skills (including edge control and flow), [2] transitions (including varied use of intricate footwork linking technical elements), [3] performance (including emotional involvement related to the music), [4] composition (including ice coverage and originality of choreography), and [5] interpretation of music (including movement and steps in time to the music and expression of the music's character or feeling).

While we will not fully describe the rubrics for scoring here, it is sufficient to note that program components and technical elements are quite clearly intended to be independent measurements. In an official ISU (2003) statement describing the IJS, the ISU claimed that the IJS would not turn skating into jumping competitions because there are other elements available to skaters.

In 2010 Olympic Winter Games, these independent judging measures were put to the test. In the men's singles event, the silver medalist landed a quadruple jump but the gold medalist only landed triple jumps, which may indicate that at that time the judging system was allowing for artistry to be the deciding factor for performances. Yet, after the second-place finisher and his national skating federation publicly criticized the judging system, the system was revised to reward quadruple jumps more (Fyodorov, 2010). This high-profile criticism made us wonder if the intentions of the IJS scoring system would be adhered to, or if it might result in a difficulty bias towards athletes who employ more technically difficult elements. In short, would the TES scores affect PCS scores even though they are meant to be evaluated independently? A difficulty bias occurs when judges tend to award higher PCS to athletes who perform more technically difficult elements, even when PCS should be evaluated independently of any technical considerations.

1.3 Further explanation of IJS scoring using Nathan Chen's 2019 ISU World Championships results

As a motivating example, consider Nathan Chen's first place performance in the Men's Singles event at the 2019 ISU World Championships (ISU, 2019). His official scores sheet from his free skate program reports that his jump elements were Quadruple Lutz, Quadruple Flip, Quadruple Toe Loop, Triple Axel, Combination Quadruple Toe Loop and Triple Toe Loop, Combination Triple Lutz and Triple Toe Loop, and Combination Triple Flip with single Euler and Triple Salchow. Moreover, his choice of having jumps for 7 out of 12 executed elements does satisfy the well-balanced program criteria for the men's free skate. A detailed explanation of how his TES was calculated is provided in Appendix A.

Within the IJS, there is similarly a fixed ceiling on scores based on the technical difficulty of the elements for each skater's program. For each element, there is an assigned base value for the element without considering quality of execution. When taking into account the judges' ratings of quality, a maximum and minimum score per element can be computed. For most elements, this maximum score is 150% of the element's base value, and the minimum score is 50% of the base value. An exception is that for the second step sequence in the men's free skate, the maximum and minimums are defined differently as reported in Table 1. In this performance, Chen achieved the highest theoretical base values for his spins and steps scores within the TES, so there is a fixed ceiling for those scores. Yet for jumps, there has not yet been a skater who has achieved the highest theoretical base value for the jumps performed in a program.

As shown in the second-to-last column in Table 1, the scores that Chen earned for these seven jump elements comprised 96.25 points out of his 121.24 point TES, so the jump scores accounted for 79% of his TES. The spins accounted for 12% and the steps accounted for 9% of his TES. The jump scores are also 44.5% of the TSS score 216.02 points. Also in Table 1, we show Chen's 2019 World Figure Skating Championships earned scores for each technical element in the second-to-last column. In the last column, we calculated the percentage of the maximum score that Chen earned. Overall, Chen's earned scores are 85% of the maximum possible scores for each element.

Table 1
Nathan Chen's 2019 World Figure Skating Championships Elements

		Base Values (BV)		Maximum Element Score		Minimum Element Score		Nathan's Earned Scores		
Element Category	Element Listing	Per-element BV	Category BV Sum	Per-element Max Score	Category Max Score	Per-element Min Score	Category Min Score	Nathan's Element Score	Nathan's Category Score	Nathan's (Earned/Max) Score
Spins	Spin 1	3.2	10.2	4.8	15.3	1.6	5.1	4.43	14.28	92.3%
	Spin 2	3.5		5.25		1.75		4.85		92.4%
	Spin 3	3.5		5.25		1.75		5		95.2%
Steps	Steps 1	3.9	6.9	5.85	11.35	1.95	2.45	5.57	10.71	95.2%
	Steps 2	3		5.5		0.5		5.14		93.5%
Jumps	Jump 1	11.5	77.29	17.5	115.94	5.75	38.65	16.26	96.25	94.3%
	Jump 2	11		16.5		5.5		13.04		79.0%
	Jump 3	9.5		14.25		4.75		12.89		90.5%
	Jump 4	8		12		4		10.97		91.4%
	Jump 5	15.07		22.61		7.54		18.46		81.7%
	Jump 6	11.11		16.67		5.56		13.22		79.3%
	Jump 7	11.11		16.67		5.56		11.41		68.5%
	Base Value Sum		**94.39**		**142.59**		**46.2**		**121.24**	**85.0%**

However, we demonstrate in our results section that PCS scores are highly correlated to TES, so jumps effectively contribute well over 44.5% of the total score.

1.4 Pairs illustration of jumps being a determining factor in overall outcome

The scores of the top three pairs free skates at the 2019 World Figure Skating Championships are reported in Table 2 (ISU 2019). The Jumps Scores and the Lifts Scores are reported, and they are subsets of the TES. Notice that the Jumps Score, TES, and PCS decrease monotonically as rank increases. Due to differences in base values on the Scale of Values, lifts in pairs programs have the potential to contribute more points than jumps to the TES.

2 Table 2

3 2019 World Figure Skating Championships Pairs Scores

Free Skate Rank	Pairs Skaters' Names	Nationality	Jumps Score	Lifts Score	TES	PCS
1	Wenjing Sui & Cong Han	China	29.58	43.84	76.84	76.84
2	Evgenia Tarasova & Vladimir Morozov	Russia	23.7	34.19	73.42	73.42
3	Vanessa James & Morgan Cipres	France	19.33	34.47	72.41	74.11

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2 Data and model

For this study, we relied upon data from Oi's (2021) Stats on Ice website, which contains the largest public database of US Figure Skating and International Skating Union competition results. The 208 worldwide competitions analyzed include major and minor competitions such as the ISU Grand Prix series, ISU competitions hosted by other countries' national governing bodies, and US Figure Skating Regionals, Sectionals, and Nationals. We used the programming language Python to the parse the data from this database, and we used the software Tableau to visualize the data and perform the computations. While the database itself contains competitions from as early as 2005, we chose to include only competition results during the 2018-2019 season, since that was the most recent complete season before the COVID-19 pandemic. In addition, because the ISU typically issues rules changes between seasons, limiting data to a single season allowed us to compare data with a consistent set of rules and scoring rubrics. The number of competitions and individual skating programs analyzed for each skating discipline appears in Table 3.

Table 3
Number of programs in data set for each skating discipline

Skating Discipline	Number of Competitions	Number of Skating Programs
Singles	120	19124
Ice Dance	53	3168
Pairs	35	636
Sum	**208**	**22928**

The data set we used included skating programs from all levels of skating from beginner to elite, which allowed us to analyze the largest number of skating programs possible from the 2018-19 season. Each particular skating discipline level is distinguished by such factors as program time length, allowed technical elements, program components scored, and sometimes age. While there are minor differences in scoring between the individual levels, our analysis shows that the overall trends remain the same.

3 Results

3.1 Jump scores contributing to Total Element Scores

The jumps' contribution to the TES will be henceforth denoted as the "Jump Score," and it is a subset of the TES. Expanding upon Chen's jump score percentages, we computed average jump score percentages of TES for all Men's and Ladies competitions within our data set and obtained the results listed in Table 4. (In 2022, the event previously called "Ladies" was changed by the ISU to be named "Women").

Table 4
2018-19 Season: Jump scores' contributions to TES

Segment	Men's	Ladies	Pairs
Free Skate	62.7%	51.6%	31.0%
Short program	53.3%	42.8%	24.8%

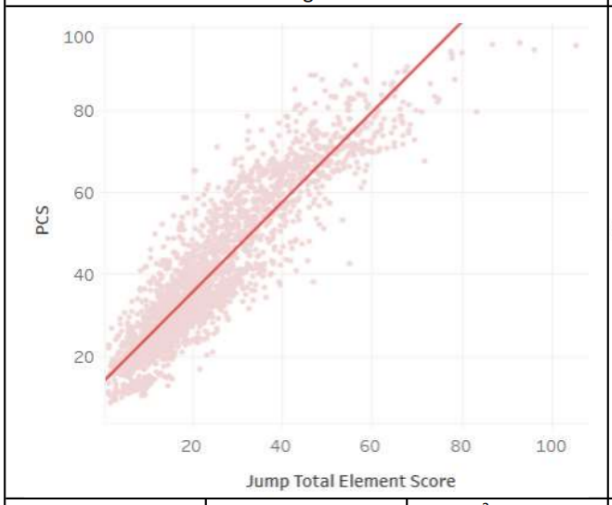
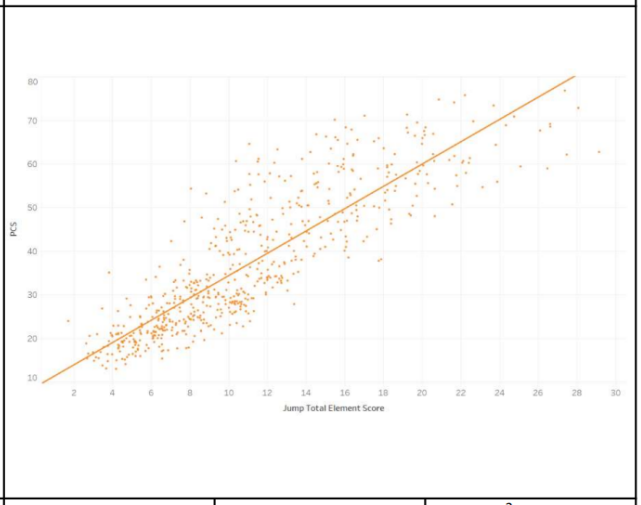
Our findings reveal that a disproportionate amount of the TES comes from the Jump Score in both Men's and Ladies skating. Free Skate programs draw higher jump score percentages than Short Programs, because the Free Skate is longer and more jumps are allowed there. However, it is interesting to note that the jump score percentages were noticeably higher for men than for ladies. More men performing quadruple jumps than ladies, and, at the lower levels of competition, it is common to see men rotating more revolutions in their jumps than ladies at the same level. Even so, 42.8% of the TES for Ladies' Short program came from jumps.

The element category that contributes the most to Pairs skaters' TES is not jumps, but rather lifts (where the lifting partner has fully extended arms and gives the lifted partner assistance to be suspended above the lifting partner's shoulder): 40.52% of the free skate score and 39.63% of the short program score is comprised of the lifts scores. However, we were still interested in focusing on examining jumps within pairs because of the ISU (2003) statement cited earlier in this paper.

3.2 Jump scores predicting Program Component Scores

While the IJS intends for all of the TES to be independent scores from the PCS, we found strong relationships between these scores, as shown in Table 5. Specifically, skaters who earned higher Jump Scores tended to also receive higher PCS. Linear regression was used on this data set since it was the regression of best fit.

Table 5
Linear Regressions with Jump Scores predicting PCS in Singles and Pairs

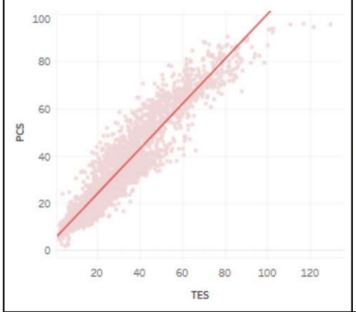
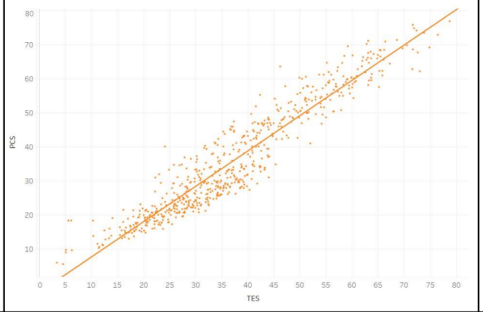
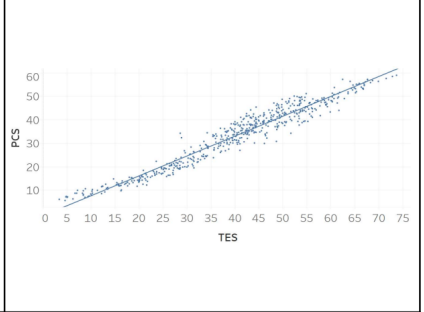
Singles			Pairs		
					
Slope	Y-Intercept	R^2 Value	Slope	Y-Intercept	R^2 Value
1.098	13.534	0.825	2.561	8.819	0.746

The high R^2 values shows that one can predict a skater's Program Component Scores from their Jump Scores with a fairly high degree of certainty, which suggests a difficulty bias towards Jump Scores. The pairs data showed weaker correlations, possibly because jumps comprise a lower percentage of the TES in pairs than they do in singles skating, as shown by comparing the data in Tables 4 and 5.

3.3 Total Element Scores predicting Program Component Scores

The results reported in Table 6 show the strongest evidence that there is a difficulty bias appearing in our data set. Within every discipline of figure skating, there is a linear relationship between the TES and PCS, whereby the R^2 value is greater or equal to 0.889. As such, the TES and PCS are not independent measures.

Table 6
Linear Regressions with TES predicting PCS in Singles, Pairs, and Ice Dance

Singles			Pairs			Ice Dance		
								
Slope	Y-Intercept	R^2 Value	Slope	Y-Intercept	R^2 Value	Slope	Y-Intercept	R^2 Value
0.955	4.647	0.889	1.083	-2.860	0.900	0.847	0.929	0.944

The slopes of the linear regressions can be revealing. All of the slopes are positive, indicating that for each TES point earned, the PCS is correspondingly increased. For skaters, this could be helpful because if they are able to increase their TES through additional training, then based on previous seasons' results, there is some likelihood that their PCS will also increase.

In Ice Dance, even though there are no jumps, the correlation between PCS and TES still remains, as shown in Table 6. In fact, the $R^2 = 0.944$ in Ice Dance is the highest correlation of all disciplines when comparing the TES vs. PCS linear regressions. This may be surprising from a spectator's perspective, since ice dance is the discipline that appears the most artistic compared to singles or pairs skating.

4 Gaining Strategic Advantage in Ice Dance

Any skaters in any discipline can use the results from the above analysis. We provide an example of how one set of skaters could benefit from the ability to predict scores.

Alexei Kiliakov and Elizabeth Tkachenko are an ice dance couple who competed internationally at the Junior level representing the United States in the 2019-2020 season. Before competing in Junior Ice Dance, Kiliakov & Tkachenko competed quite successfully at the Novice level in 2019, culminating in winning the US national championships at the novice level (US Figure Skating, 2019). Before skaters move up to a higher level for the following season, it would be helpful for coaches to

hypothesize how athletes might have ranked in the current if they had competed in that higher level. There are slight differences in the composition of the programs at the Novice and Junior levels: two additional elements are required at the Junior level, and these elements present additional point-earning possibilities.

For the purposes of this analysis, we assume that the judges' scores for the two additional elements are at base value. Also, in this analysis, we assume that Kiliakov & Tkachenko might earn the same score on the Novice-level choreographic step sequence (not a required element at the Junior level) as the Junior one-foot step sequence (not a required element at the Novice level) since these two elements have the same base value. Table 7 shows this information.

Table 7

Calculation of Novice competitors' hypothetical scores in free dance at the Junior level

Element Listing	Kiliakov & Tkachenko's Actual Novice Score	Predicted Junior score
Twizzles	6.09	6.09
Circular Steps 3	9.11	9.11
Combination Spin 4	6.9	6.9
Straight Line Lift 4	5.89	5.89
Curve Lift 4	5.65	5.65
Choreo Step 1 (Novice) /One Foot steps (Junior)	4.63	4.63
Choreographic Element		1.1
Choreographic Element		1.1
Total Element Score	**38.27**	**40.47**
Predicted Program Component Score		**35.22**
Predicted Junior Free Dance Total Segment Score		**75.69**

This predicted score would have placed Kiliakov & Tkachenko in tenth place for the free dance during the 2019 season. Of course, these predictions do not account for improvements from future training, but they are still useful for competitive insight. The competitive history for this couple after 2019 shows that they ranked 7th in both the 2020 and 2021 US National Championships at the junior level.

In some situations, an international analysis of scores such as this can help inform skaters of their choice of home country. A couple ranked 7th in the United States is unlikely to be selected to represent the country at higher level international competitions, because each county can only select two or three representatives per discipline. Kiliakov & Tkachenko elected to represent the nation of Israel in the 2021-22 season, and as such, they had the opportunity to compete at the highest international junior level competition, the 2022 World Junior Figure Skating Championships.

5 Conclusion and Discussion

In this article, we performed a quantitative analysis of IJS scoring analysis comparing TES and PCS. By analyzing 2018-2019 season data, we show that in spite of the ISU's efforts to separate out TES and PCS, the TES and PCS are closely correlated across every skating discipline and that there are indeed difficulty biases within figure skating scores. Furthermore, in spite of any Well-Balanced Program requirements, there is a strong imbalance in scoring that renders jumps as a strong predictor of success in both singles and pairs skating. Future studies could examine the potential interplay between technical and program component scores further by measuring the extent to which credentialed judges agree on scoring of one of these two scores when the other score is held constant or is given: for example, how likely are two skaters who earn the same TES to earn the same PCS?

From an athlete's perspective, these results mean that most training emphasis should be on technical aspects of skating, which would mean jumps for singles and pairs. From a sport governance perspective, the main way to eliminate these imbalances and correlations would be to change the scoring tables to de-emphasize jumps, and to have the judging panels score program components more independently from technical elements (e.g., to have separate officials score TES and PCS). Moreover, by understanding these relationships, skaters can easily decide which elements to focus on and even predict their competition scores with reasonable certainty. From a coach's perspective, the linear regressions presented here could be used to assess readiness for their athletes to compete at higher levels of skating.

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