Mathematics and Sports

A model-based analysis of master swimmers performances

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Abstract

We analyse the qualifying times introduced by World Aquatics for the masters' World Championships. Our approach is based on the theory we proposed for the fair scoring of athletic performances, where we claimed that the scoring should be directly linked to the energetic cost of the effort. The application of that approach led to the realisation that the evolution of performances with age follows a well-defined, "universal", curve, practically rectilinear in the "young" and "old" branches. We show in this paper that the same approach, based on energetic cost and the universal curve, can be applied to swimming. Age adjustment factors can be derived simply within this framework, and they compare favourably with the empirical ones. We analyse the World Aquatics qualifying times, showing an overall consistency but also identifying a few discrepancies. A new set of qualifying times, computed with our methods, is also proposed as a proof-of-concept.

Keywords: Swimming, master athletes, qualifying times, energetic cost

1 Introduction

The masters' movement in sports is a rather recent phenomenon, linked to the gradual improvement of the situation of the elderly over the last century [Toynbee 1948]. Thanks to the practice of sport, many inconveniences related to old age, although not totally avoidable, may be reduced and delayed. Veteran sports materialise the slogan of "sport of all", joining generations of athletes. The presence of master athletes in every discipline is more and more important, with numbers often exceeding those of the younger sportsmen. Curiously the masters' age groups are not populated by "retired" champions, although some are present. New champions do appear, many of whom have started their involvement with competitive sports at a mature age. As Kusy and Zielinski [Kusy 2006] point out, "outstanding veteran athletes are usually athletes who did not win the most prestigious international trophies when young".

Master's sports are globally coordinated by the International Masters Games Association [IMGA], founded in 1995, which organises, every four years, the World Masters Games. The latter is a multisport event comprising some 30 individual and team disciplines. Many international sports federations have their world and/or continental masters' championships, reserved to athletes above a given age minimum. In the case of swimming, which will be the object of this work, the first world championships took place already in 1986 [Masters Archive]. For World Aquatics, master swimmers must have a minimum age of 25. The athletes compete within their age group, which spans five years. Thus the age groups are 25–29, 30–34, and so on, with five-year age groups as high as necessary. The last age group for which records have been registered is that of 100–104.

World Masters swimming championships have been an enormous success and this has led the international federation to introduce some strict rules [Masters 2023]. First, the total number of competitors is limited to 6000. In fact, the entries are closed even before the announced deadline if this limit is reached. This is meant to keep the competition manageable. (And the athletes cannot enter more than five individual events). Second, the entry times must have been obtained during a sanctioned meet and should not exceed a fixed qualifying time. This is meant to eliminate frivolous entries and keep the competition high-level. Moreover, the qualifying times are not only used in controlling the entries. If the time of the event swum exceeds the qualifying standards, the time is not shown in the results sheets and is noted as "no time", meaning that the swimmer does not participate in the official classification. Moreover swimmers who clearly fail to achieve the qualifying time may be excluded from the upcoming individual events.

These rigorous rules, which can have serious consequences for the swimmers, put an extra onus on the determination of the qualifying times. In this work, we are going to examine the qualifying times introduced by World Aquatics for the World Masters Championships of 2023 (which are valid also for the 2024 championships [Masters 2024]). Unfortunately, the author is not privy to the method World

Aquatics used to obtain the qualifying times. Thus we will use the methodology introduced in previous works of ours, based on mathematical modelling, to analyse said times. (In this work we shall limit ourselves to freestyle events). In what follows we shall start with the general theory of the evolution of performances with time, initially developed in a different context, and we shall show how it can be adapted to swimming. Next, we shall introduce age adjustment factors, and compare our approach to existing proposals. Focusing on the world masters championships qualifying times, we shall use the age factors to highlight existing discrepancies. Finally, we shall propose a simple approach that allows us to derive consistent qualifying times.

2 Evolution of performances with age

Curiously, participation in an intense physical activity past a certain age, let's say once physical maturity is reached, is a human-only trait: none is observed in ageing animals. However, it is a fact that even highly trained master athletes experience a decline in their performances after the first few decades of life. This of course varies with the activity in question, which is why different sports have different age thresholds for their masters' age groups.

There is no scarcity of articles devoted to the study of the evolution of swimming performances with age. The classic study is that of Tanaka and Seals [Seals 1997] who performed an analysis of the top performances from the US Masters Swimming Championships focusing on the (freestyle) 1500 m. They found the times increased linearly from peak levels at circa 35-40 years of age until 70 years of age and increased exponentially thereafter. More recent studies have essentially confirmed those results. Reaburn and Dascombe [Reaburn 2008] analyse the factors influencing the decrease in performances. Curiously, while commenting on Tanake and Seals results, they affirm that the declines are curvilinear from the age of 35 up to approximately 60-70 years of age, while the former authors speak about a linear decrease. An intriguing result is a figure based on the results of Trappe [Trappe 2007], where the authors are talking about speed while presenting the running pace (which is essentially the inverse of velocity) and which, given that the overall distance is fixed (in this case a marathon) is just proportional to time. Rubin and Rahe [Rubin 2010] refer to early studies which found a yearly increase in times of about 1 % for most swimming events, an estimate refined, thanks to later studies, shown to be about 0.6 % per year up to an age of 70. Rubin and collaborators [Rubin 2013] addressed the same question based on results obtained from their longitudinal studies of the performances of champion master swimmers.

Before going further let us present the result most authors have been working with, describing it as "linear" and "exponential". Figure 1 shows the world record times [WA Records] over 1500 m for men and women as a function of age.

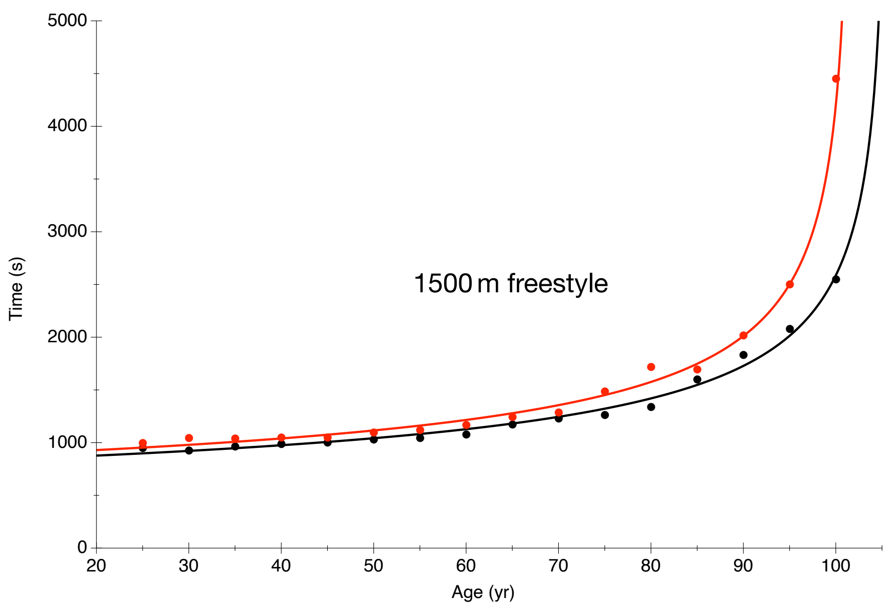

Figure 1. *World record times over 1500 m freestyle for men (black circles) and women (red circles) as a function of age. The fitted line will be explained later in the article.*

In [Grammaticos 2007] we introduced an alternate approach, namely the physical basis for the scoring of athletic performance, where we argued that the physical quantity that should be used to assess performance is the free energy, i.e. the amount of energy that can be converted into work. Subsequently, we addressed the question of the evolution of performances with age [Grammaticos 2009] aiming at proposing a fair scoring system for master athletes. We found that the evolution of performances of a particular cohort (in many cases the world record holders) followed a characteristic curve, with a shape independent of the event under consideration. Moreover, the curve could describe equally well the evolution of the performances of young athletes. A schematic representation of what we came to call the universal curve is shown in Figure 2.

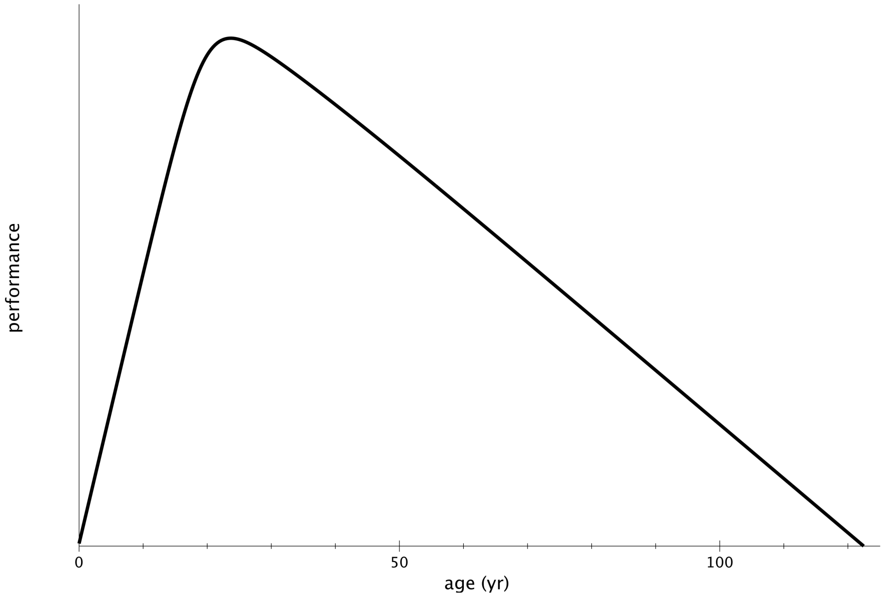

Figure 2. *The evolution of performance with age.*

Two remarks are in order at this point. First, the slopes of the two practically rectilinear branches of the universal curve have a ratio of roughly 5. This means that the progression of young athletes is 5 times faster than the decline of the masters. The second point concerns the zeros of the curve. Before discussing it we should point out that the universal curve does not describe the career of a given individual. An athlete picks up a given discipline at some age, practices during a certain period (at which time his performances can be described by a segment of the universal curve), and then stops. The universal curve describes the evolution of the performances of a large cohort, ideally of the whole human race. To perform one must be alive. So, the curve starts from 0 at birth and goes again to 0 at the moment of death. Thus the zero of the universal curve at a large value of age, can be an indicator of the maximal human life span. The analysis of athletics performances yielded values in the 120–130 bracket [Grammaticos 2020a]. By looking at how the human race performs with advancing age, we can obtain an estimate of the possible life duration. G. West [West 2017] addressed the same question from a physicist's point of view and concluded that the maximum human life span was around 125 years, in excellent agreement with the one obtained from the analysis of athletic performances.

The important question now is: do the performances in swimming follow the universal curve? To address this question we must first determine swimming's energetic cost. There is abundant literature on this point but few results propose quantitative approaches that can lend themselves to mathematical modelling. The total resistance F met by the swimmer, is a function of the swimmer's velocity. Were the resistance constant, the total energy spent to cover a distance s would be given by the product Fs, and the energetic cost ε which is just the energy spent per unit time, would be given by the product Fv where v is the velocity. For a velocity-dependent resistance, the energetic cost of an effort of duration t is

$$\varepsilon = \frac{1}{t}\int_0^t F(v)v(\tau)d\tau. \tag{1}$$

If F were precisely known as a function of the velocity it would have been straightforward to express the energetic cost of swimming. In the simplest case, one expects a power-law type relation of the form

$$\varepsilon = Cv^{\gamma}. \tag{2}$$

The scoring tables [WA Scoring] proposed by World Aquatics are based on a simple formula where the number of points is proportional to the cube of the mean velocity, assuming thus that the resistance to the swimmer's progression is due to turbulent flow, proportional to the square of the velocity. We turned thus to the literature hoping to obtain a more refined estimate. Alas, most authors contend themselves with graphic representations without proposing analytical fits. A notable exception is the works of diPrampero, in particular his paper [di Prampero 2019] on the "Energy cost of human locomotion on land and water" in collaboration with Osgnach. In that work, the authors present their own results on the energetic cost of swimming and complement them with results obtained by other teams. In particular, analysing the results of Capelln et al. [Capellin 1998] one obtains the following dependence of energetic cost on velocity

$$\varepsilon \propto v^{2.6}. \tag{3}$$

The team of diPrampero [di Prampero 1974] measured the energetic cost of crawl swimming using a special experimental setup and obtained for the velocity dependence of the drag coefficient the expression $F \propto v^{1.2}$ which would lead to an energetic cost of

$$\varepsilon \propto v^{2.2}. \tag{4}$$

An analysis of Zamparo et al. results [Zamparo 2020] would lead to values close to those of Capellin et al. We see that the values of the exponent γ obtained by the various teams are in the middle-low 2 zone, significantly lower than the value of 3 suggested by the turbulent hydrodynamic resistance. One can understand this by the fact that the swimmer's body is not passively towed in the water but, by the mechanism of self-propulsion, adapts its position and buoyancy resulting in lower values of γ.

In what follows we shall assume that the dependence of the energetic cost on the velocity is given by a simple power law with an exponent

$$\gamma = 2.5, \tag{5}$$

but it must be clear that any value in the 2.2-2.8 bracket would not modify appreciably our conclusions. In Figure 3 we present below, we plot the velocity to the power γ for the masters' 100 m freestyle world records [WA Records] as a function of age, separately for men and women.

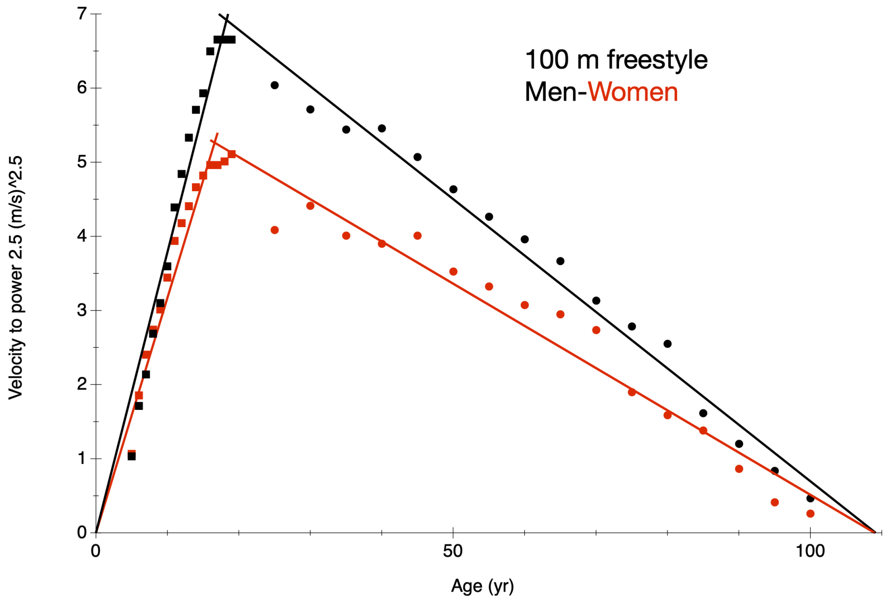

Figure 3. *Velocity to the power 2.5 as a function of age for men's (black) and women's (red) world records.*

First, we observe that the rectilinear dependence of a quantity proportional to the energetic cost on age, one expects, if the universal curve were to be valid in the domain of swimming, is quite close to reality. Both curves are of the type $v^{2.5} = c(Z - A)$ where A is the age of the swimmer and Z is an upper limit in years close to 110 for both sexes. This value is somewhat smaller than the one obtained in the case of athletics which was in the bracket of 120–130 years. A possible explanation for this is that Swimming is a sport much "younger" than Athletics and has a wider margin of progress particularly in the masters' groups. Should the progression of masters' swimming continue, we believe that the upper limit Z will increase getting closer to the value obtained in Athletics.

Once the masters' branch of the universal curve was obtained we asked the question whether the observation concerning the young swimmers' branch was valid here also. To check this we plotted a line with a slope five times that of the masters' starting from age zero. We drew the velocity to power γ of the best young swimmer performances on top of this line, which is supposed to fit them. We remark that the agreement is satisfactory. (A little bit less so in the case of women, but this is true for both branches of the universal curve. It is perhaps a manifestation of a persistence of an absence of parity between men and women both in young and master age groups).

An interesting remark is in order here. In Figure 1 we have fitted the master records of the 1500 m freestyle with a continuous curve. Now is the time to explain its origin. Given that we expect all master swimming records to follow a branch of the universal curve described by $v^{2.5} = c(Z - A)$ we can use this expression to obtain the dependence of time on age. We find an expression $t = b/(Z - A)^{0.4}$ and use the data on the records to fit the parameters of the expression. The result is the two continuous curves which are in agreement with the data, confirming thus the validity of the choice of the value 2.5 for the exponent γ.

3 Introducing age adjustment factors

One question that arises naturally is how the performances of older athletes compare with those of the younger ones, taking into account the natural decline due to age. The answer to the question is the age adjustment factors. Applying such a factor one can, starting from the performance of, say, a 60-year-old swimmer obtain the equivalent performance, for instance, of a 25-year-old. The question has been addressed for swimming and different solutions have been suggested.

The first and older one is what is known under the moniker "Finnish Formula", created in 1995 in Finland [Finnish formula]. It has a simple expression

$$f=\sqrt{\frac{Z^2-A^2}{Z^2-N^2}} \tag{6}$$

where A is the age of the swimmer, Z is a cut-off age, fixed at 98, and N is chosen equal to 25, i.e. the lower age limit for masters. Thus the adjustment factor is equal to 1 for $A=25$. The Finnish Formula ceases to be valid beyond $A=Z$, the latter being fixed at 98. However this is not something that cannot be remedied: it suffices to increase the cut-off value to one beyond the last recorded master world record, for instance, 105. Further tweaks of the formula are also possible by taking an exponent different from 2.

A. Rawson [Rawson 2018] proposed a more complex solution to the same problem aiming at remedying the limitations of the Finnish Formula. His adjustment factor has the form of a 6th-degree polynomial

$$f=\sum_{n=0}^{6} c_n A^6 \tag{7}$$

and the seven coefficients c_n are optimised for each event, separately for men and women. An example of the coefficients of (7) (for the women's 50 m freestyle event) is

$$\begin{aligned}
c_6 &= 0.000000000055826\\
c_5 &= -0.000000019980038\\
c_4 &= 0.000002807675499\\
c_3 &= -0.000197478468079\\
c_2 &= 0.007186097374188\\
c_1 &= -0.128590589172152\\
c_0 &= 1.893216340550950
\end{aligned}$$

In [Grammaticos 2020b] we introduced age adjustment factors for Athletics based on the fact that masters' performances follow the universal curve. In a nutshell, if the energetic cost follows the straight line $Z-A$ it suffices to divide by this quantity to obtain a constant value, independent of A. Here we apply this adjustment, based on the universal curve, to swimming. As shown in the previous section, the evolution of times with age follows in the mean the curve $t=b/(Z-A)^{0.4}$. It suffices thus to divide by such a factor to adjust the performances.

A comparison of the three age factors is given in figure 4.

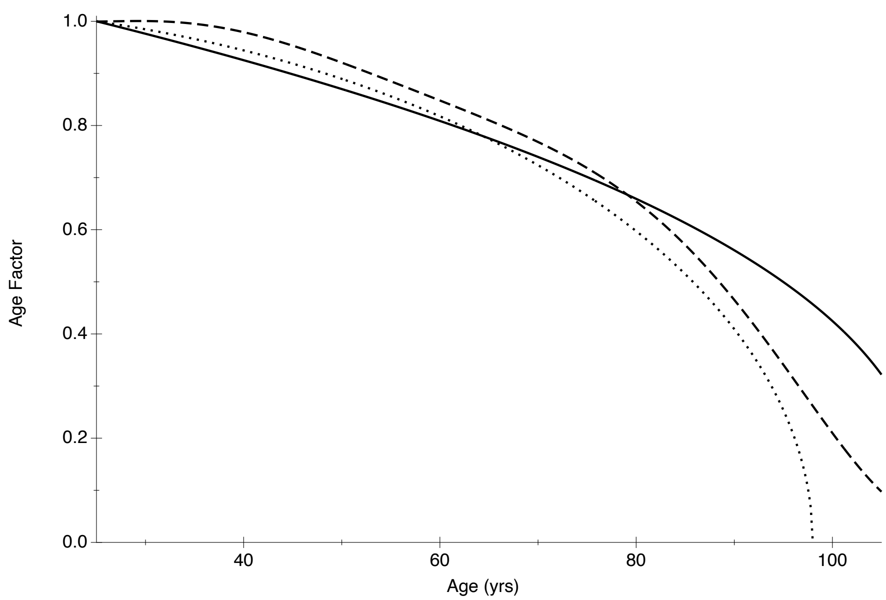

Figure 4. *Comparison of the three age factors as a function of age: Finnish (dots), Rawson (dashes), based on the universal curve (continuous line).*

We remark that the behaviour of the three is quite similar. The main advantage of the one proposed by the author is that it is based on a theory rather than having been solely empirically introduced.

4 The World Aquatics masters' qualifying standards

As we explained in the introduction, to keep masters' championships manageable and of a high level, World Aquatics has introduced qualifying standards in the form of times to be realised before the championships in official competitions and which cannot be exceeded during the championships, lest the swimmer be removed from the official classification. The qualifying times are given below for men

MEN Age	50 m	100 m	200 m	400 m	800 m
25	29.40	1:03.15	2:20.60	5:01.95	10:38.55
30	30.10	1:04.35	2:24.05	5:09.85	10:43.50
35	31.10	1:05.35	2:26.50	5:17.80	10:59.35
40	32.50	1:07.30	2:31.45	5:26.70	11:08.25
45	33.50	1:10.80	2:36.40	5:39.55	11:33.00
50	34.80	1:14.55	2:45.35	5:46.50	12:12.60
55	36.30	1:17.20	2:59.20	6:06.30	13:02.10
60	38.30	1:22.15	3:09.10	6:38.95	14:01.50
65	40.80	1:28.10	3:23.95	7:10.65	15:00.90
70	43.80	1:37.00	3:42.75	7:55.20	16:30.00
75	49.00	1:48.90	4:07.50	8:34.80	17:59.10
80	1:02.00	2:00.75	4:25.35	9:44.10	19:38.10
85	1:12.00	2:22.05	5:06.90	10:53.40	22:16.50
90	1:22.00	2:58.20	6:16.20	11:52.80	25:44.40

and for women

WOMEN Age	50 m	100 m	200 m	400 m	800 m
25	34.00	1:11.30	2:38.40	5:38.60	11:52.80
30	35.00	1:14.25	2:43.35	5:48.50	12:12.60
35	36.00	1:16.25	2:51.30	6:03.35	12:32.40
40	38.00	1:19.20	2:58.20	6:11.25	13:12.00
45	40.00	1:25.15	3:13.05	6:36.00	14:21.30
50	42.60	1:31.10	3:27.90	7:10.65	15:20.70
55	45.50	1:36.05	3:37.80	7:40.35	16:10.20
60	48.00	1:41.00	3:52.65	8:05.10	17:14.55
65	51.80	1:50.90	4:11.45	8:49.65	18:28.80
70	56.00	1:56.80	4:23.35	9:34.20	19:57.90
75	1:00.50	2:10.70	4:54.05	10:38.55	22:56.10
80	1:10.00	2:33.45	5:56.40	12:12.60	24:45.00
85	1:31.00	2:58.20	6:40.95	13:41.70	27:53.10
90	1:52.00	3:32.85	7:55.20	15:40.50	33:00.00

The first consistency check of the qualifying times is to verify that they follow the masters' branch of the universal curve. We thus plotted the velocity, obtained from the qualifying times, elevated to a power of 2.5, as a function of age. In Figure 5 we present the result for men, together with the best fit with an expression $c(Z-A)$ where Z has been fixed at $Z=95$ for all five events.

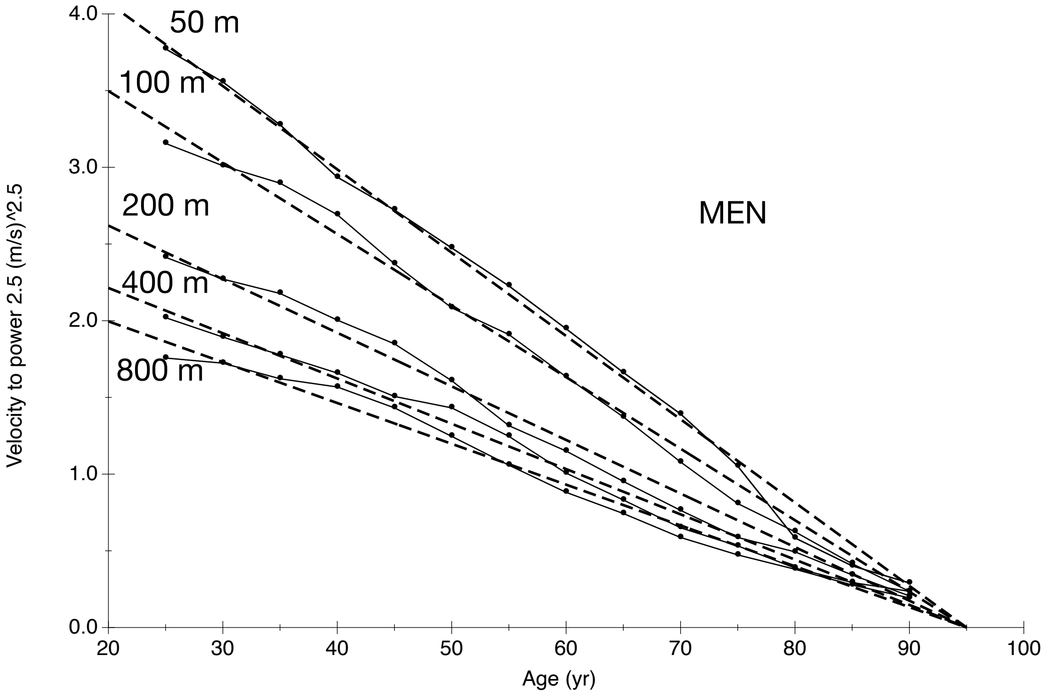

Figure 5. *Qualifying velocities elevated to a power of 2.5 for men.*

We remark that the qualifying times, most probably empirically derived, follow roughly the behaviour expected, i.e. the quantity $v^{2.5}$ decreases linearly with age, defining the downgoing branch of the universal curve.

The corresponding women's results are shown in Figure 6.

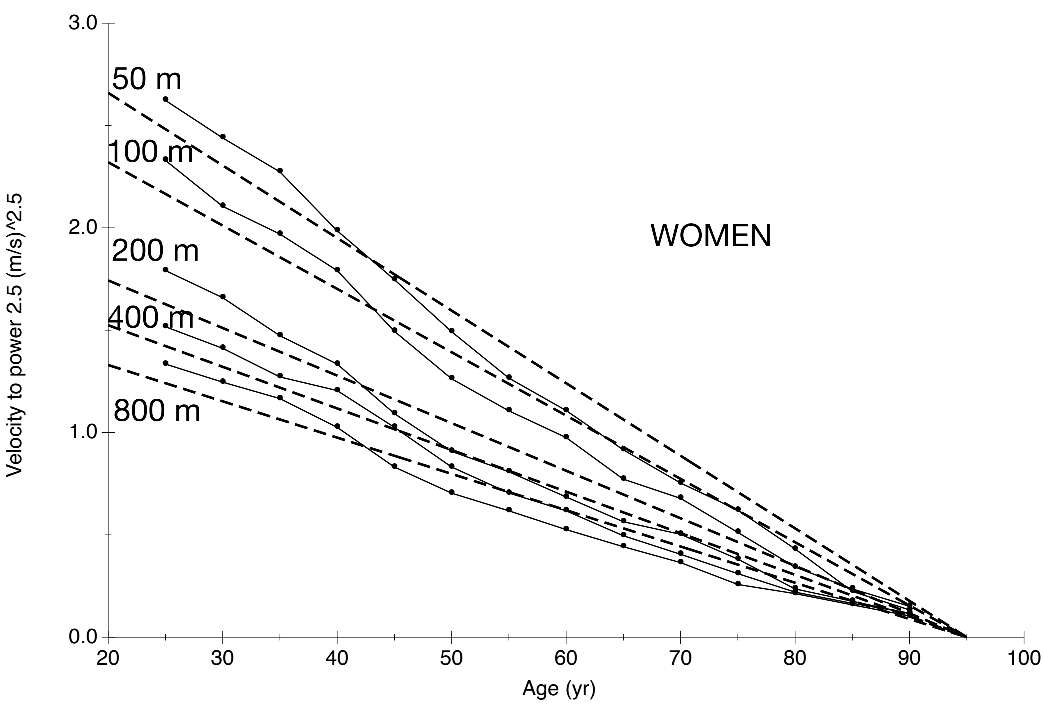

Figure 6. *Qualifying velocities elevated to a power of 2.5 for women.*

Here the deviations from the predicted linear dependence are more pronounced than in the case of men. One possible explanation could be a relative paucity of statistics for women.

While we consider that the first consistency check is satisfied, it is clear that a more detailed analysis is necessary to appraise the quality of the qualifying times proposed. Had the quantities $v^{2.5}$ followed precisely the straight line predicted by the universal curve, it would have sufficed to divide by a quantity proportional to $(Z-A)$ in order to see them aligned on a horizontal line. So, to highlight the deviations, we multiplied the quantities $v^{2.5}$ obtained from the qualifying times by the quantity $75/(95-A)$. The results in the case of men are shown in Figure 7 while those for women are displayed in Figure 8.

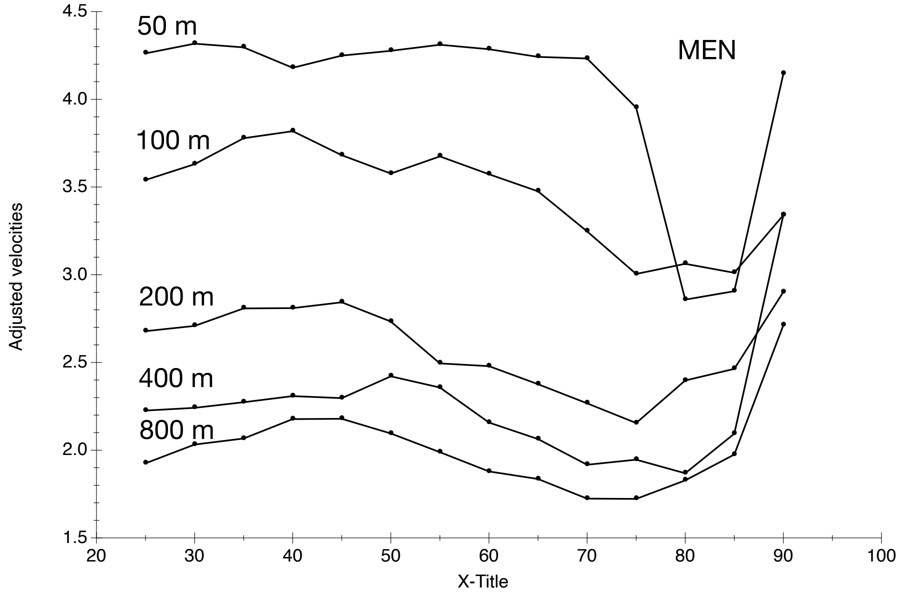

Figure 7. *Adjusted velocities for men.*

One first remark is that the points are definitely not aligning along a horizontal line. Moreover, in the case of the 50 m, there is a noticeable dip at ages 80 and 85, which signals the inadequacy of the qualifying times. Indeed while the time difference from $A=70$ to $A=75$ is just 5.2 s, between $A=75$ and $A=80$ there is a huge gap of 13 s. It goes down to 10 s between $A=80$ and $A=85$. Also, the point for $A=90$ in 400 m coincides with that of 100 m, again an indication that something went wrong. A glimpse at the qualifying times suffices to confirm that 11:52.80 for 400 m is precisely 4 times 2:58.20, the qualifying time for 100 m.

In the case of women's qualifying times,

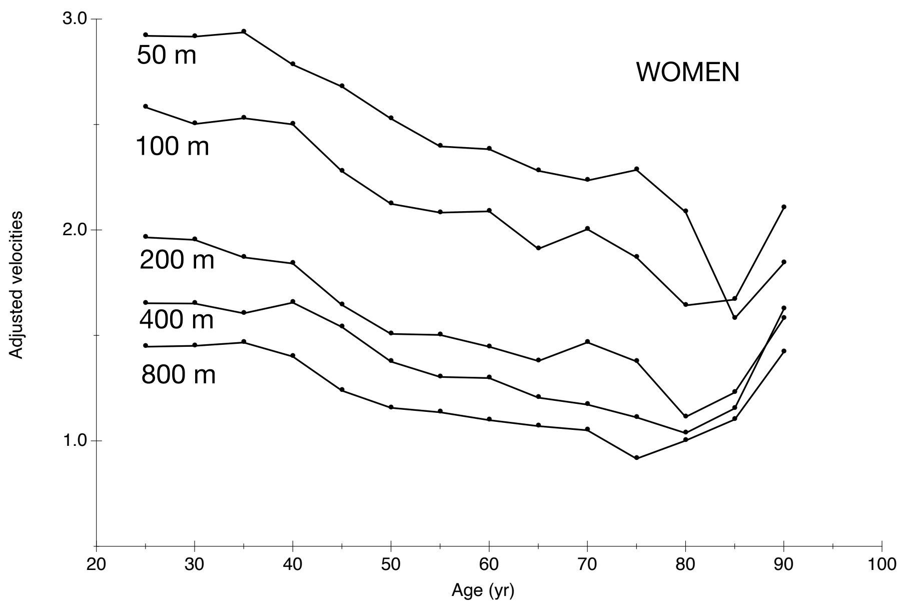

Figure 8. *Adjusted velocities for women.*

there are again some line crossings, indicating improperly estimated qualifying times. Moreover, there is a global negative slope of the line joining the points, meaning that, in the case of women, the qualifying times are based on empirical data reflecting a lower quality of older age performances.

In case someone wonders why all lines, for both men and women, end with an upward slope from $A=85$ to $A=90$, we believe that this is an artefact due to the cutoff age $Z=95$, which is too close to 90. Had we chosen a larger value for the cutoff, this effect would have been dampened, but at the price of a somewhat worse fit in Figures 5 and 6.

Given the situation with the qualifying times imposed by World Aquatics, one can wonder whether there is a possibility to propose a somewhat improved set. It turns out that, if one uses the framework we have presented in the previous sections, based on the theory of energetic cost and the universal curve, such a proposal is possible. Assuming that the times we are seeking can be derived by a direct application of this theory, they should conform, as explained in the previous section, to the expression $t = b/(Z-A)^{0.4}$. Given the choice $Z = 95$, there remains only one parameter, b, to be determined for each event, for men and for women. We decided that since we believe that the World Aquatics qualifying times have, the discrepancies notwithstanding, a solid empirical foundation, to use an anchor point obtained from said times. We had thus fixed the value of b in such a way as to have the value of the expression $t = b/(Z-A)^{0.4}$ coincide with the known qualifying time for $A = 50$. Once this single parameter is fixed, the qualifying times are obtained straightforwardly. In the Table below below we give the list of the qualifying times we obtained for men, which we computed following the method just described.

MEN **Age**	50 m	100 m	200 m	400 m	800 m
25	29.16	1:02.47	2:18.56	4:50.36	10:13.92
30	30.30	1:04.35	2:22.73	4:59.10	10:32.39
35	31.10	1:06.44	2:27.37	5:08.83	10:52.96
40	32.11	1:08.79	2:32.59	5:19.77	11:16.90
45	33.36	1:11.47	2:38.52	5:32.20	11:42.36
50	34.80	1:14.55	2:45.34	5:46.50	12:12.60
55	36.47	1:18.14	2:53.32	6:03.21	12:47.94
60	38.48	1:22.43	3:02.83	6:23.14	13:30.70
65	40.92	1:27.67	3:14.46	6:47.51	14:21.59
70	44.20	1:34.30	3:29.17	7:18.34	15:26.77
75	48.13	1:43.11	3:48.70	7:59.26	16:53.30
80	54.00	1:55.69	4:16.59	8:57.71	18:56.88
85	1:03.51	2:16.60	5:01.77	10:32.39	22:17.60
90	1:23.80	2:59.53	6:38.19	13:54.44	29:24.26

We remark that the abnormal gap in the case of the 50 m has disappeared. Moreover, the computed qualifying time of the 400 m for $A = 90$ is substantially larger than four times the qualifying time for the 100 m for the same age. While these are positive developments, the approach is far from perfect. A global remark concerning the computed times is that the qualifying times for $A = 90$ are systematically larger than the ones of the official qualifying times and, in fact, excessively large, in the case of 800 m. We believe that this is due to the cutoff age Z taken equal to 95, which leads to too large a correction for $A = 90$. Moreover, the computed qualifying times for the younger age groups tend to be smaller than the initial ones, but this is probably not an undesirable feature.

The computed qualifying times for women are presented in the Table below.

WOMEN **Age**	50 m	100 m	200 m	400 m	800 m
25	35.69	1:16.34	2:54.22	6:00.88	12:51.54
30	36.77	1:18.63	2:59.46	6:11.74	13:14.76
35	37.96	1:21.19	3:05.30	6:23.83	13:40.62
40	39.31	1:24.70	3:11.86	6:37.43	14:09.68
45	40.84	1:27.34	3:19.32	6:52.87	14:42.70
50	42.60	1:31.10	3:27.90	7:10.65	15:20.69
55	44.65	1:35.49	3:37.92	7:31.42	16:05.11
60	47.10	1:40.73	3:49.88	7:56.19	16:58.60
65	50.10	1:47.14	4:04.50	8:26.47	18:02.81
70	53.89	1:55.24	4:23.00	9:04.79	19:24.73
75	58.92	2:06.00	4:47.55	9:55.65	21:13.47
80	1:06.10	2:21.37	5:22.62	11:08.30	23:48.78
85	1:17.74	2:46.26	6:19.43	13:05.97	28:00.36
90	1:42.59	3:39.38	8:20.66	17:17.10	36:57.25

The situation in the case of women's computed qualifying times is different from that of the men's. Here the computed qualifying times for the younger age groups are systematically larger than the official ones. This is because, as pointed out when commenting on Figure 8, the initially proposed qualifying times tended to favour the more advanced ages. Once this effect is removed, due to the new parametrisation, the resulting times for the younger age groups are increased. Of course, the remark concerning the age group $A = 90$ is valid here as well. This could be remedied by pushing the cutoff age to larger values, perhaps up to $Z = 105$–110, since 100–104 is the last age group for which world records have been registered.

The question that naturally arises at this point is whether the set of computed qualifying times is offered as a replacement for the ones proposed by World Aquatics. While we believe that our approach corrects the minor discrepancies in the set-up of the qualifying times we do not envision it as a viable alternative to the existing ones. They are proposed mainly as a proof of concept, aiming at demonstrating that it is possible, by using a mathematical model of the energetics of swimming and the evolution with age, to propose a consistent set with just a minimum of an empirical input. We shall return to this point in the conclusion.

5 Conclusion

This paper aimed to study the evolution of the performances of master swimmers with advancing age. Our approach was motivated by the World Aquatics policy to introduce qualifying times for the Masters' World Championships. The method we based our study on was one proposed in the domain of Athletics. As we have argued in previous publications of ours, the way to obtain a fair scoring of athletic performance is through the consideration of its energetic cost. This approach not only offers a natural way of scoring but also allows one to study the evolution of the performances with age. In the case of Athletics, we showed that the masters' world records, when drawn as a function of age, follow essentially a straight line over several decades of age. Moreover the evolution of the young athletes' records also follows a straight line. The union of these two branches (the slopes of which have a factor of roughly 5) form what we dubbed the universal curve. As we showed in studies in the domain of Athletics this is true for both men's and women's records.

The first question addressed in this paper was whether the results obtained in Athletics could be transposed to the case of Aquatics. The main difficulty faced in such an approach was the knowledge of the energetic cost of swimming. While several teams have performed studies of energy expenditure during swimming, very few results that could lend themselves to a mathematical modelling approach exist. This is understandable since the experimental setting for such studies is quite complicated and it is difficult to obtain results over a large range of velocities: the swimming style ends up degrading both at excessively low and demandingly high velocities. Using a simple ansatz for the energetic cost of swimming we have studied the evolution with age of masters swimming world records. We have limited ourselves to freestyle swimming but we do not expect our results to depend crucially on this choice. The important finding was that the evolution of swimming records with age, for both young and old swimmers, follows the universal curve, when one uses the energetic cost for the ordinate of the curve, confirming the conclusion reached in the case of Athletics. One question that is invariably asked for master performances is how the latter compare to those of younger athletes. To answer this question, age adjustment factors were introduced, allowing to convert a performance obtained at some age to an equivalent one for a different, younger or older, age.

As already explained, the kindle for this work was the existence of qualifying times for the masters' swimming championships. We have shown that the qualifying times follow, grosso modo, the masters' branch of the universal curve, albeit with notable deviations. A more detailed study of the qualifying times using an age adjustment approach pinpointed discrepancies most probably due to errors during their derivation. One way to avoid such a situation is to use the methods presented in this paper to consistently derive the qualifying times. We have shown how such a method would work in practice, although our results were meant as an illustration rather than a concrete proposal of a new set of times. It is clear that to be able to propose a realistic set of qualifying times one should work with a more precise expression of the energetic cost, consider a cut-off age around or even beyond the one for which world records exist, and use an empirically fixed anchor point based on a large set of existing competition results. The present work sets the framework for such a future implementation.

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